RIGA TECHNICAL UNIVERSITY

Monte Carlo Methods in Financial Engineering

1. Module

Fundamental Principles of Monte Carlo Methods

Literature

1. Paul Glasserman: **Monte Carlo Methods in Financial Engineering**. Series: Stochastic Modelling and Applied Probability. Springer-Verlag New York, Inc., 2004.
2. Malvin HH. Kalos, Paula A. Whitlock: Monte Carlo Methods. Wiley-VCH Verlag GmbH&Co.KGaA, 2004.
3. Martha L. Abell, James P. Braselton, Lorraine M. Braselton: A Beginner's Guide to Mathematica. 2002.

What is Monte Carlo?

- The term «Monte Carlo» was first assigned to the class of mathematical methods by scientists working on nuclear weapon at Los Alamos in the 1940s.
- The essence of the method – an invention of gambling, the behavior and outcome of which can be used to investigate an interesting phenomenon.
- Here are two examples of what we understand and do not associate with Monte Carlo calculations (next slides)

What is Monte Carlo?

Example 1.

A circle inscribed inside a square. $\frac{S_{circle}}{S_{square}} = \frac{\pi}{4}$. If we randomly place points within the square, the fraction $\frac{\pi}{4}$ will also represent the portion of points that fall within the circle. You can estimate $\frac{\pi}{4}$ using the following method:

- Place a circular pan with a diameter of L inside a square pan with sides of length L, and collect items in both pans;
- Generate random pairs of points in the Cartesian coordinate system in such a way that the points fall within the square and count the number of points that fall within the circle.

Through many experiments, it has been determined that the value of this ratio is close to $\frac{\pi}{4}$, and this is referred to as an estimate for $\frac{\pi}{4}$.

What is Monte Carlo?

Example 1.

By performing 1 000 000 experiments, there is a very high probability (95% confidence) that the number of points inside the circle will fall within interval of 784 600 to 786 200.

This gives an estimate for $\frac{\pi}{4}$, which falls between 0.7846 and 0.7862;

When we compare it to actual value of $\frac{\pi}{4}$, which is 0.785398.

What is Monte Carlo?

Example 1.

- This demonstrates that random sampling can be used in solving mathematical problems. In this case, it would be the estimation of a definite integral $\int_0^1 \int_0^{\sqrt{1-x^2}} dxdy$.
- The obtained results have a statistical nature, and they are obey to random laws.
- Determining the value of π is, of course, faster and more accurate with non-Monte Carlo methods. However, in multidimensional cases, Monte Carlo methods are often the only effective means for evaluating integrals.

What is Monte Carlo?

Example 2.

Let’s assume someone wants to estimate the chances of winning a game of *solitaire*, and they have a perfectly shuffled deck of 52 cards before dealing the cards.

Once a specific strategy for moving cards from one pile to another is chosen, the problem becomes a part of elementary probability theory.

- Create a program that generates randomly shuffled variations of the 52-card deck, prepare scenarios from different card arrangements, and then simulate playing until the end of the game.
- The observations from many repetitions of these simulations would lead to a Monte Carlo evaluation of the likelihood of success.

What is Monte Carlo?

- Monte Carlo method is a technique that involves the deliberate use of random numbers in calculations, which exhibit a stochastic process structure.
- By a stochastic process, we mean a sequence of states whose dynamics are determined by random events.

History of Monte Carlo

1777 Compte de Buffon: If needle of length L are thrown onto a plane covered with parallel lines at a distance of d from each other (where d > L), then the probability P of a needle crossing one of these lines is $P = \frac{2L}{\pi d}$.

Laplace: This could be used to calculate π (inefficiently).

1930s The first significant scientific application of Monte Carlo: Enrico Fermi used it to model the movement of neutrons in fissile materials. Serge: «Fermi delighted in astonishing his Roman colleagues with the results of his «too-good-to-be-true» predictions from his experiments.»

History of Monte Carlo

1940s The name «**Monte Carlo**» was introduced by Nicholas Metropolis and Stanislaw Ulam.

Ulam was involved in nuclear weapons projects, which were classified, so the work required a coded name. This name refers to the *Monte Carlo Casino* in Monaco, where Ulam's uncle often gambled.

History of Monte Carlo

1953	The algorithm for generating samples for different probability densities was developed by Metropolis, Rosenbluth, Teller (Hastings generalized in 1970).
1962, 1974	The first *Quasi-MC* calculations by Kalos, Levesque, Verlet.
1965	The first *Variational-MC* calculations by Bill McMillan.

Monte Carlo Methods in Financial Engineering

- Monte Carlo simulations are a significant tool in the pricing and risk management of derivative securities.
- Prerequisites: knowledge of mathematical tools typically used for continuous-time financial model specification and analysis.
  - Ito calculations;
  - Stochastic differential equations;
  - Martingales;
  - Girsanov's theorem statement.

Thank You!